Saturn ' s Ring

The problem I am going to explain is the instability of Saturn's ring.
First thing first, let's have a look at the characteristics of this amazing system.
Galileo is the first human on the earth to observe the rings with a telescope. Then Robert Hooke. Then Cassini found the gaps thus the first one is called the Cassini Division. Nowadays, with more powerfull telescopes and flying by detectors, we have gained more details.
Most of the features can be summarized into this chart. Since Maxwell or rather to say Jeffreys has already ruled out gas or liquid structure with very simple arguements, the following work i am going to show is base on a particle ring model. The particle size is from micrometers to meters. However, the distribution of them is unknown. So we have to guass! Generally, the ingrediant is ice with little dirts. The orbital of it is about 7000km to 80000km above saturn's equaor. An amaziong thing is that it is possess a thickness of meters. A near look at it reveals many fine structures, for example ... (next slide)
B ring has a mysterious spokes structure. We have got a gif animation here. There are also disturbance caused by moons. The animation here. Two moons induced a complex wave in ringlets. To have a relatively complete discription, we should check other wave band of light. The photo down here is a comparision of visiable light a calculate radio wave image. However, we need more close observations etc.. Anyway, there are many more work to do. So our knowledge of Saturn's ring is far from complete.
In the following theoretical view, these three things are the most concerned. Namely size, distribution and a few fine structures.
Lets have a brain storm. I listed some of the characteristcs on the left side and physical process on the right. We have tidal force here. Tidal force tears objects into piece if it goes too high. So it might be a constrain on the maximum size. Gravitation and angular momentum conservation tells us that the system has to be a thin plate. Why? check the origin of a solar system. Friction and collison means this is a disipative system and more precisely we will treat it as a near equilibrium system. Power law might gives us a more complex structure.
We will go to the details. To save time, I won't explain the formulas in detail. If u want to know the details ask me at the end of this talk.
As we said, the maximum size is determined by the tidal force. The concept we are going to show here is similar to Roche limit. Inside a particle, there are two dominating forces. Tidal force and the tensile force. The tidal force trys to tear it into pieces while the tensile force hold different parts of the particle together. Finally, if the particle remains a whole, the tearing tidal force should be smaller than the tensile force. The equation is shown here. With the data observed, the final answer is that the radius of a particle at a orbital radius r should be less than this. Since the radius of the particles is about this range, we can draw a conclusion that the radius of the particles should be less than 100km. Fits well with the observations.
Lets move to the size distribution.
The size distribution is roughly an assumption! Usually we use a power law form. It reads... (read the words on the screen i.e. the constrains)
We won't talk too much on the thin ring phemonenon 'cause it is simple. To calculate the thickness, viral theorem is applyed and we get a answer of this.
Here comes the most important part in this talk - the instability or phenomenally [fi'n■min■li]the ringlets.
We hope we can derive some equations that can show the rings can split into small ringlets. Physically, instability or ringlets phenomenon may mean that particles flow to dense place. Or in another way, the dense the number of particles the more particles comes. Thus if we give a little perturbation to the system, the perturbation grows! Finally, the dense place possess all the particles. Thus a gap formed. This is not a steady system tranditionally. This is a near equilibruim system. This is all the things I am trying to tell u. As the slides goes, this will be more clear.
Next we will define some quantities. Drift flux........   Introduce power law hypothesis.
Let look more into the defination. If we write it in to this form, ...(a form with B)........ This indicates that, (read the below part). Instability exists!
But the meaning of the formula is not clear enough too! We hope to find a relation between of more foundamental

parameters. Or we want to discribe the instability with observed quantities. Then we should do more calculation and finally we will get this long eq.. The equation tells us that ...(read the below parts).......
δ<0 for τ>1 => unstable
δ<-1 for τ<<1 => unstable
for sufficiently low Σ (or τ), g must vanish with Σ because other parameters in g can not go to infinity. => ∂g/∂Σ>0 => stable
for sufficiently high Σ (or τ), g goes to infinity with Σ because other parameters can not go to zero. => ∂g/∂Σ>0 => stable

To be accurate, other parameters is used to derive equations. Lin did more precise word. Will not be shown in the slides.

To show the theory, Lin et al. did numerical experiments.
(Fig. 1 & Fig. 2)
meaning of symbols:
τ=ΣA, Σ is related to δ (power-law exponent)
$\dot{N} = \dot{n}/(3\pi\Sigma\nu)$, $\dot{n} = \frac{-2}{\Omega R}\,\partial g/\partial R$, flux of particle flow (minus means inward)
$H = (R/R_0)^{1/2}$, in which R_0 is difined as $\tau(R_0)$=1. (We can alway normalize the observed τ, i.e., optical depth)
a is the initial and b is the end. six curves separated by equal time intervals. total simulated time is e-folding time. (can not be longer than this because the numerical truncation errors grow!!!)
Explanations of the two figures:
Fig. 1 shows the evolution of a smal perturbation when δ=0 (means Σ decrease with R). redefined $\dot{N}$ becomes small as time goes on.
Fig. 2 is a unstable condition.
Both of the simulations have no cutoff even they need one when Σ is too large because power-law is assumed for $\sigma \propto \Sigma^{\delta}$. But this does not affect the trend that $\dot{N}$ evolves. Another important argue is that $\dot{N}$ is the flux of particle flow. Even $\dot{N}$ is cut when it is too high, particles still flow to other places as long as $\dot{N}$ is nonzero.
The most important thing we know from the theory is that ringlets can form from a slightly perturbated ring through viscous interaction in a Keplerian particles disk. Fits well with our physics consideration.
Simple discription. Actually this leads to less dissipation or in other words less production of entropy.
(
Spoke structure is not explained because they used a axisymmetric perturbation in the beginning.
In those work, they did not give the origin of the power law.
)
(Figure 2 gives us an impression that the flow of particle may exist forever! That is because this simulation is done under perturbation and Lin did not give cutoff to the system! Physics concerning gives a tendency that when the particles becomes very little, the flow trend to stop. Or more academicaly, the calculation is done near the equilibrium!)

(The persistency of the ringlets is discussed in Lin's paper.)

(Observation show that at the gaps τ<<1. This indicates that δ<1 in these regions.(Check out the power law assumption.))

(Since dissipation always exist, the source of the energy must be explained or the system would die (crash to Saturn) as time go on. That is another work, namely the lifetime of Saturn's ring.)